DOING MATHEMATICS IN PROFESSIONAL DEVELOPMENT: THEORIZING TEACHER LEARNING WITH AND THROUGH SOCIOMATHEMATICAL NORMS

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This research report discusses the work guiding Researching Mathematics Leader Learning (RMLL) (1), a 5-year research project intended to investigate mathematics PD leaders' understandings and practices associated with developing mathematically rich learning environments. In this report, we will focus on how we are elaborating the construct of sociomathematical norms with leaders as a way to understand the workings of a mathematically rich environment. Distinguishing social norms from SM norms is an important dimension of our work with leaders. We have learned to attend to the language that we use carefully and to pay attention to how the leaders themselves engage in mathematics in order to understand how they make sense of the construct of SM norms.

Few take issue with the fact that high quality teachers make a difference in children's learning, so it stands to reason that having high quality professional educators (i.e., leaders) is important for teachers' learning. How we engage in mathematical work with teachers in professional development (PD) provides significant models for teachers for what it means to engage in mathematical reasoning. The PD context is one place where teachers have opportunities to learn what it's like to develop mathematical habits of mind such as generalizing, proving, engaging in argumentation, and connecting representations to their symbolic equivalents. Yet there is a dearth of research on how we structure and lead mathematical work with teachers (Even et al., 2003; Stein et al, 1999). And we have much to learn about how engagement with mathematical reasoning in PD helps teachers learn how to create such environments for their students and to think about the mathematical work that gets accomplished in the classroom. Cohen & Ball (1999) note that, "ironically, while the role of the teacher educator is critical to any effort to change the landscape of professional development… there is little professional development for professional developers" (p. 26). In a study of the California Mathematics Projects, Wilson and Berne's (1999) review suggested that the mathematics often gets negotiated away in professional development. This may be a direct result of the lack of PD for leaders.

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rich environment. (In the context of our work, we use the term PD leaders because they are often teachers who also do professional development work rather than district level professional developers or university personnel.)

Sociomathematical Norms in Professional Development

Sociomathematical (SM) norms are the specific ways students (translated to PD—teacher participants) engage in mathematical work in the classroom (or PD). These norms govern the ways people engage with mathematics (Yackel & Cobb, 1996). SM norms include things like what counts as an acceptable mathematical explanation or what constitutes a mathematical justification. These norms are negotiated either explicitly or implicitly by teachers and students (in PD, leaders and participants) through social interactions. To build SM norms more likely to promote mathematical understanding, researchers suggest that teachers need to pay attention to the mathematical work of their students (Kazemi & Franke, 2003; Sherin, 2001). Similarly, our focus is on how leaders learn to pay attention to participants' mathematical thinking and how their learning connects to fostering productive SM norms in mathematics PD.

Exploring mathematics with groups of people is inherently a cultural practice. As communities of people work together over time, normative behaviors and practices are developed. Several factors shape these practices—the students, the specific contexts, the mathematical content, as well as what is valued and defined as competent participation in mathematics class (Lampert, 2001). The same must also be true for PD practices. Wilson and Berne (1999) and Lord (1994) suggest that what is typically defined as competent participation in PD is engaging colleagues in social pleasantries and avoiding dialogue that may prove uncomfortable for participants. Moreover, Remillard and Rickard (2001) suggested that teachers inquiring into practice and digging into mathematical ideas were not typical norms in the PD seminars they facilitated unless teachers were provided support and scaffolding. While social norms, such as being polite and considerate of colleagues, are often the focus of teachers and PD facilitators, they do not expressly support the deepening of mathematical knowledge.

The negotiation of SM norms is part of enculturating participants into particular ways of reasoning in mathematics. These ways of doing mathematics may, or may not, be compatible with participants' ways of knowing and being in the world. As a result, facilitators must be cognizant of the tensions that may exist between the cultural ways of engaging in the world that participants bring to learning opportunities and the practices practitioners and researchers believe are important for fostering deep understandings of mathematics (Forman, 2004; Ladson-Billings, 1995). An additional issue at play here is that leaders' capacity to cultivate deep understanding has everything to do with their ability to know what and how to press for mathematical understanding. Moreover, as leaders work with colleagues in PD, tension is likely to arise as they focus on teachers' mathematical understandings and uncover teachers' mathematical confusion. Leaders are also likely to grapple with navigating the use of errors, negotiating participants' social and intellectual status and connecting mathematical work done in the PD context to work that teachers do with their students.

Method

Seminars for Leaders

To provide some context for our research, we briefly summarize the PD for leaders. They participated in a series of seminars to focus their attention on SM norms and the cultivation of mathematically rich environments. The seminars were organized around a set of videocases showing facilitators leading mathematical activities with teachers. Each seminar included the following set of activities:

- Working on mathematical tasks with consideration of the mathematics in the task and how teachers might approach it
- Viewing videos of professional development involving teacher discussion of the mathematical task
- Discussing the videos to consider what mathematical explanations teachers are offering to one another and how the group engages in mathematical reasoning
- Engaging in a series of "Connecting to Practice" activities designed to help leaders apply some of the ideas in designing and facilitating mathematics professional development and to consider implications for their own work
- Reflecting through journal writing
- Doing homework activities involving examining the ideas from the seminars in leaders' own work or in their observations of other leaders.

Through these experiences, we attempted to make explicit both the mathematical content and the way teachers engage in doing mathematics. The mathematical content of the seminars emphasized algebraic ideas, particularly how to create arguments for the generalized case of a solution.

For this research report, we focus on one particular session in which leaders viewed and discussed a video from a professional development session called *Janice's Method.* In the video, Janice shares her way of solving 92-56. She solves the problem by subtracting 60 from 90 to get 30 and then adding 6 back on to get 36. As she explains how she knew to add 6 back on, she explains, "so 90 is two away from 92. So I needed to recover that two. And 56 is four away from 60, so I needed to recover my four." The facilitator in the video case presses Janice to explain what she means by "recovery" and a discussion unfolds in the group to discuss why Janice's method works and how it can be generalized to other subtraction situations as well as discussions about what subtraction means as an operation. This videocase is used because it is an opportunity to discuss the facilitator's role in opening up discussion to develop conceptual explanations.

Participants

Data from this report come from two different groups of participants. The first group was a highly experienced group of teacher leaders (n=11) with whom we piloted our materials (will be referred to as the pilot group). The pilot group was most experienced participating in and facilitating the *Developing Mathematical Ideas* (Schifter, Bastable, Russell, 1999) materials, typically with seven to nine years of experience. The second group was a less experienced group of teacher leaders (n=13) working in a district in the southwestern United States, typically with one to three years of experience facilitating PD (will be referred to as the SW group). The SW group had also participated in professional development often tied to the district math curriculum, assessment, and a range of other activities such as mathematics study groups and lesson study. The vast majority of leaders in both groups had experience teaching elementary school. A few had experience in middle or high school. The pilot group worked with the *Janice's Method* once (as the third in a sequence of four videocases over 2 months) while the SW group worked with it twice, once at the beginning and once at the end of their participation (in a sequence of eight videocases over the course of an academic year).

Data Sources and Analysis

We documented the leaders' experiences in the seminars by videotaping and transcribing each session with two video cameras, which enabled us to capture both whole group discussion and at least two small group discussions for each activity. All work generated by participants were kept in a journal and used for analysis. Participants also completed on-line surveys before and after the seminars to provide us with both demographic information and to reflect on their experiences. For the purposes of this report, we focused our analyses on one session, *Janice's Method*, common to both the pilot group and the SW group. Qualitative data analytic techniques were used to compare the perspectives and engagement of the two groups.

Findings

Our elaboration of SM norms in the PD context has led us to focus, as initial entry into these ideas, on the nature of explanations. Our deliberations about how to focus on adults' ways of engaging with mathematics have led us to a two-tiered approach. First, we pose questions that focus leaders on describing in detail (1) what mathematical explanations were offered in the group regardless of how and who did the explaining. With that discussion as a foundation, we look again at the case materials to deliberate (2) how the group being observed engaged in mathematical explanation. This second question then allows us to press leaders to pay attention to the nature of questioning, the treatment of errors and confusion, the locus and distribution of kinds of mathematical questioning, and the consequences of presenting particular explanations or raising particular questions to the groups' mathematical work (in other words, what mathematical work did the group accomplish). Our focus on the nature of explanations has led us to unpack four relevant practices: (1) sharing, (2) justifying, (3) responding to confusion and errors, and (4) questioning (see Table 1).

Distinguishing social norms from SM norms is an important dimension of our work with leaders. We have learned to attend to the language that we use carefully and to pay attention to how the leaders themselves engage in mathematics in order to understand how they make sense of the construct of SM norms. Our analyses thus far have led to the following two assertions.

1. *The pilot group's greater experience participating in and facilitating PD focused on mathematical reasoning and students' thinking was reflected in the way they engaged in the mathematical tasks themselves and how they analyzed the videocase with respect to SM norms.*

   There were important differences between the way the pilot group and the SW group engaged in the mathematical task itself during the leader seminar. In preparation for viewing the videocase of *Janice's Method*, both groups did the mental math task, 92-56 themselves.

After generating various methods for solving the problem, the leaders were asked to think about how the solutions worked and what connections they could see between them. Two things happened among the pilot group. One small group felt that they were so familiar or "saturated" with this math task that they could not engage in discussing and comparing the mathematical solutions. This seemed a potential hazard of doing mathematics that is quite familiar to leaders. Others, however, in the pilot group engaged with the mathematics for its own sake. They used the solutions generated by the group as a springboard for further mathematical inquiry. For example, they investigated how to model subtraction as an operation (as removal versus difference) on the number line and whether their approaches would work with decimals. In contrast, the SW group used their small group time to connect the solutions they shared as a group back to the work of classroom teachers or what students might do or struggle with. We considered why we might be seeing this difference between the pilot and SW group and conjecture that it might have to do with how leaders interpreted the intellectual work of doing mathematics in PD. To press for explanation and justification among colleagues may involve issues of risk (not wanting to convey any confusion or misunderstanding) or an expectation that everyone can naturally deduce the mathematical significance of solutions. As one participant explained to her small group, "When Cathy (Carroll) asked me how I did it, I wasn't expecting to explain how. I thought that once I explained what I did, I had already done it. I wasn't thinking like I would think [about] it when I was teaching. If I was teaching, I would have prepared for my explanation in some way." This same participant reflected later in the session noted that she has been tentative to push mathematical ideas when facilitating PD. We found that our explicit attention to how teachers engaged in mathematical reasoning collectively in the video allowed leaders to think about the mathematical purpose of highlighting, selecting, and sequencing particular solutions in a group discussion (Stein et al., 2006).

2. *There was growth in the SW leader group's attention to how participants' in the videocases engaged in mathematical discussions. However, learning to elaborate the ideas behind sociomathematical norms is not a trivial endeavor.*

We engaged in lengthy discussions in our research team about when and how to introduce the idea of SM norms. Although Cobb, Yackel and colleagues draw attention to things such as what counts as an acceptable or sufficient explanation, we discovered that even experienced leaders react to these terms in ways that might obscure their intent. For example, as we discovered in piloting, the words *acceptable* and *sufficient* led some leaders to focus on the absolute correctness or validity of solutions rather than on what becomes negotiated in a particular context. This may be due to the everyday understandings of the terms acceptable and sufficient.

The term *"norm"* turned leaders' attention to social norms that are general to any group that is trying to work together. The SW group for example generated the following list of group norms they thought were established in the *Janice's Method* videocase: not interrupting, valuing process, no cellphones, no sidebars, open to hearing ideas. One participant began her small group's discussion of what group norms may have been in play in the videocase by stating, "When we set our norms, we draw a picture of Norm. Is that what you guys do?" to which others nodded. Social norms, are of course, important in understanding how a group functions, but they do not necessarily give much purchase in thinking about what mathematics gets accomplished and how that was achieved. However, we did not think that we could introduce the term sociomathematical norm without a set of meanings and activities to link it to. We also used the phrase “norms for mathematical reasoning” interchangeably with SM norms, thinking that the latter may be excessive jargon for a non-academic group. For this reason, after much deliberation within our own research team and discussions with the pilot group, we settled on experimenting with an initial question, “How is the group engaging in mathematical explanation?” When prompted with this question, the SW group did focus more on the mathematical work being done in the videocase. The discourse in the SW group shifted over the course of the seminar series from a more heightened focus on the ideas of individuals in the videocase (and whether they did or did not understanding something) to more concern with how teachers in the videocase engaged with one another’s mathematical thinking. We then attempted to link their discussions about how the group engaged in mathematical thinking with questions about how those discussions provided evidence for the norms for mathematical reasoning (or SM norms) that were established or being negotiated in the videocase. We are now in the process of analyzing how the groups made sense of the idea of SM norms, how they connected their ideas to their discussions about how teachers engage in mathematical reasoning, and what impact their thinking about SM norms had on their planning for and facilitation of PD.

Table 1: Norms for Explanation That Support Teacher Learning in Professional Development

	Productive Social Norms	Productive Sociomathematical Norms
Sharing	• Teachers and the PD leader listen respectfully to one another • Teacher share solutions or strategies • Teachers work together to find solutions to problems • Multiple solutions may be explored	• Sharing has a purpose of extending and deepening mathematical thinking • Sharing consists of explanations that emphasize the meaning of mathematical ideas • Mathematical connections among solutions, approaches, or representations are explored
Justifying	• Teachers describe and give reasons for their thinking	• Justifications consist of a mathematical argument • Justifications emphasize why and how methods work
Questioning	• Both teachers and the PD leader pose questions • Questions support multiple voices and ideas	• Questions push on deepening understanding of mathematical ideas
Confusion/ Error	• Confusion and error are accepted as part of the learning process • Teachers are not put “on the spot” over incorrect answers • PD leader encourages teachers to clarify their explanations	• Confusion and error are embraced as opportunities to deepen mathematical understanding—comparing ideas, re-conceptualizing problems, explore contradictions, pursue alternative strategies

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Conclusion

For both the pilot group and the SW group, introducing and making sense of sociomathematical norms allowed them to think explicitly about how teachers engaged in mathematics during PD. How leaders engage with and take up the idea of SM norms is naturally tied to their understandings of the purpose of doing mathematics in PD (see also Elliott & Kazemi, 2007). We have reported here the beginning of our analyses of how leaders made sense of collective mathematical activity. How leaders take these ideas up as they facilitate PD is a major concern of our research project and will be the focus of future analyses connecting their participation in these leader seminars to their own work as PD facilitators.

Endnote

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