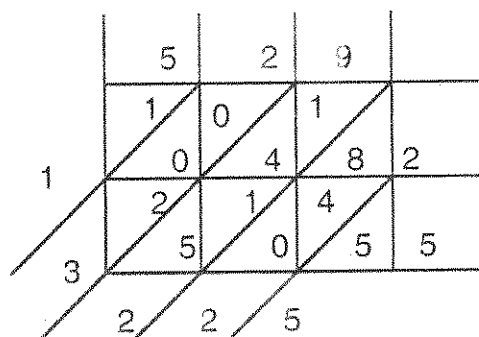
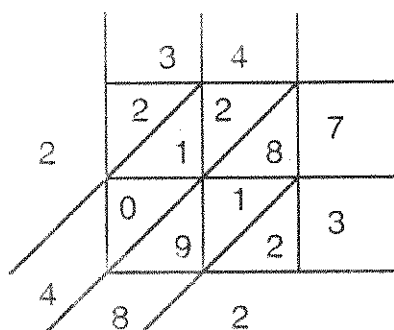


Problems on the exemption test will be modelled closely on a selection from the following problems, but will *NOT* be identical to them: Numbers and/or contexts will be changed, and aspects of different problems in this list may be mixed and matched. Also hints will be omitted, and an actual test problem could include only one or some parts of the corresponding sample problem. You may get any help you like in figuring out these sample problems, but you will not be permitted any notes or electronic devices (calculators, cell phones, etc.) during the test.

Throughout the test, “solution by diagram” means you should draw a diagram, or a sequence of diagrams, with labels and explanations so that a reader is able to see a complete solution to the problem.

Communication of mathematics is one of the main goals of Math 170. Therefore your solutions should be clear enough that someone who does not already know how to solve the problem can follow your reasoning. You must make clear not only what mathematical manipulation or calculation you are doing, but why you are doing it.

1. An airline passenger fell asleep halfway to her destination. When she awoke, the remaining distance was half the distance traveled when she was asleep. For what portion of the trip was she asleep? Hint: The answer is *not*  $1/4$ .
2. The lattice method for computing the products  $34 \times 73$  and  $529 \times 25$  are shown below. Explain why this algorithm works, in particular explaining the place value of digits entered in the lattice. Hint: It will probably be helpful to work the problems using a more familiar algorithm and compare those calculations to the lattices.



3. Give two story problems that would lead to the calculation  $4 \div \frac{2}{3}$ , one that uses the measurement model for division (also known as the repeated subtraction, or quotative, model) and one that uses the sharing model (also known as the partitive model). Label which of your problems uses “measurement” and which uses “sharing”.
4. Daryl's iPod has a total of 130 songs on it. Every song on the iPod is in exactly one of the three categories R&B, jazz, and pop. There are twice as many R&B songs as jazz songs, and 10 more pop songs than all the R&B and jazz songs together. How many songs of each type are on the iPod?
5. In base ten, you can tell whether a number is even (i.e., a multiple of 2) simply by looking at its last digit. Can you recognize an even number by its last digit in base 2? in base 3? in base 4? In each case, either explain how you can and prove it always works, or show you can't by giving both an even and an odd number with the same last digit.

6. Jo and Andy live 45 miles apart. Each left home to bicycle towards the other's house at the same time.
- Assume Jo pedals at a steady 6 miles per hour, and Andy at a steady 4 miles per hour. How far apart are they after 90 minutes? How long after they start do they meet?
  - If the next day they leave at the same time, Jo pedals at a steady 5 miles per hour, and they meet after 3 hours, how fast did Andy pedal (assuming a steady rate)?
7. Draw arrays to represent each of the following ways to compute  $29 \times 12$ , and indicate briefly where the factors and partial products in each part appear in your answers. Then explain how your arrays can be used to see that all four computations produce the same final value.
- $29 \times 12$ .
  - $20 \times 12 + 9 \times 12$ .
  - $30 \times 12 - 12$ .
  - $20 \times 10 + 9 \times 10 + 20 \times 2 + 9 \times 2$ .
8. To start the problem  $253 - 76$ , many people would write the following.

$$\begin{array}{r} 1 \quad 14 \\ 2 \quad 5 \quad 13 \\ - \quad 7 \quad 6 \\ \hline \end{array}$$

Explain the meaning of each the three appearances of the digit "1" above.

9. Consider the following problem.
- Farmer Nguyen has pigs and chickens and no other animals on his farm. He looks into his farmyard and says to his daughter, "I see nine animals with 24 legs. How many pigs and how many chickens do I see?"*
- Describe how a child could solve this problem with manipulatives (e.g., toothpicks, blocks of different colors and sizes, and/or cutouts of different shapes and sizes).
  - Give a solution by diagram for the problem. Be sure you clearly explain the reasoning that goes with the diagram. If necessary, show successive steps of constructing the diagram.
  - Give an algebraic solution that corresponds step-by-step to the solution by diagram you gave in part (b). Be sure to identify the quantities represented by any variables you use.
  - Identify all the quantities that are involved in solving this problem. Note: a quantity is something that has a numerical value. Thus "legs" is not a quantity, but "number of legs" is. However, "number of legs" is ambiguous in this problem, and requires further clarification. Check: Have you included a quantity corresponding to every number that appears in your solution?
10. Draw two diagrams that illustrate the calculation  $12 \div 3$ , one that shows the measurement model for division (also known as the repeated subtraction, or quotative, model) and one that shows the sharing model (also known as the partitive model) for division. Label which is which, explain why each diagram illustrates the particular model, and indicate clearly where the divisor 3 and the quotient 4 appear in each diagram. Hint: The diagram solutions for the measurement and sharing models for a division calculation are different.

11. Solve the following base six problem in each of the ways listed.

$$\begin{array}{r} 52_{\text{six}} \\ +34_{\text{six}} \\ \hline \end{array}$$

- (a) Pictures of (base six) manipulatives (units, longs, etc.), with clear indications of original set and how you regrouped;
- (b) Expanded algorithm for addition: the sum for each place value written on a separate line, then these partial sums added.
- (c) Or just the standard “shorthand” algorithm with marks (slashes and small digits) to show regrouping as we do in base ten – *but the digits you write must be in base six.*

(If you need to do some base ten calculations, write them off to the side and label them as base 10.)

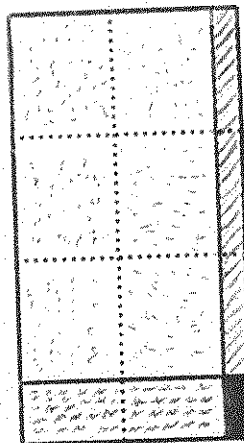
12. I have made  $5 \frac{1}{3}$  pints of strawberry jelly. My jelly jars hold  $\frac{3}{4}$  of a pint each.

- (a) How many jars do I need to hold all the jelly I have?
- (b) I will make as many full jars as I can, and put the remaining jelly in another jar. How much jelly is in the last jar?
- (c) How much of this last jar will be filled with jelly?
- (d) I will use all the full jars, and only the full jars, as gifts. How many jars do I have to use as gifts?
- (e) Draw one or more diagrams that illustrate the calculations you did to answer the questions in parts a, b, c, and d. Indicate clearly how each of the answers appears in your diagram(s). For any of the answers that is a fraction, identify the referent unit.
- (f) The four answers in parts a, b, c, and d relate to the four ways of dealing with a division that “doesn’t come out evenly”: round down, round up, leave a remainder, or convert the remainder to a fraction (that can be included in the quotient). Which is which?

13. Find the difference between  $1001001_{\text{two}}$  and  $1110111_{\text{two}}$ . Multiply  $546_{\text{seven}}$  by  $36_{\text{seven}}$ . (Hint: It may be helpful to use the lattice method from problem 2.) What number comes before  $1000_{\text{nine}}$ ? What number comes after  $4455_{\text{six}}$ ?

**Note:** To receive credit, you must show your work for the first two questions in the appropriate base, not in base ten.

14. The array diagram below represents the product of  $2\frac{1}{4} \times 3\frac{1}{2}$ . What part of the product is represented by each of the shaded regions?



15. (a) We sometimes read 3200 as “thirty-two hundred” instead of “three thousand, two hundred.” Explain why our decimal number system makes this correct. That is, why can we read “thirty” for the initial three, even though it doesn’t represent the value thirty? Hint: It may be helpful to contrast with the corresponding Roman numeral, or the Roman numeral XCXIV. (If you do not find this helpful, ignore this hint.)
- (b) Give two ways to read 1,400,000. Challenge: Give one or more additional ways to read 1,400,000.
16. Give examples of story problems leading to the calculation  $9 - 5 = 4$  involving 5 cats and using each of the following subtraction situations. For parts a, b, and c, show how each situation would be shown on the number line (using arrows between points on the line), and for part d, draw a suitable diagram.
- (a) Literal take-away (also called removal), find the difference.
- (b) Literal take-away, but the difference is given, find the subtrahend.
- (c) Missing addend.
- (d) Comparison.
17. Give two mental math strategies for computing each of the following. Challenge: Give additional ways for at least a, b, and c.
- (a)  $47 + 54$
- (b)  $64 \times 25$
- (c)  $92 - 56$
- (d)  $159 \div 13$
- (e)  $3\frac{3}{4} + \frac{2}{5} + \frac{1}{4} - 1\frac{1}{5}$
- (f) 3800 minus 9% of 3800
18. Make an argument using arrays to prove you can find the product of a pair of positive numbers by doubling one factor, halving the other, and then multiplying. Your reasoning must show that this works for all pairs of positive numbers.
19. (The following problem is from an SAT exam, but requires only fifth grade math and careful reading.<sup>1</sup>)
- A flock of geese on a pond were being observed continuously.
- At 1:00 p.m.,  $\frac{1}{5}$  of the geese flew away.
- At 2:00 p.m.,  $\frac{1}{8}$  of the geese that remained flew away.
- At 3:00 p.m., 3 times as many geese as had flown away at 1:00 p.m. flew away, leaving 28 geese on the pond. At no other time did any geese arrive or fly away or die. How many geese were in the original flock?

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<sup>1</sup>Dancis, J. (no date), [http://www-users.math.umd.edu/~jnd/subhome/Reading\\_Instruction.htm](http://www-users.math.umd.edu/~jnd/subhome/Reading_Instruction.htm), August 16, 2010.